Thermal scaling in the three-dimensional Ising model

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Nuclear multifragmentation is a process occurring at the limits of nuclear excitation, and, as such, portrays an appropriate richness and complexity. While the fundamental problem of dynamics vs. statistics is still debated, it appears ever more clearly that many thermal/statistical features underlie the empirical body of data. In particular, "thermal scaling" has been seen in a large set of data [1–3]. The three-dimensional Ising model was chosen to study thermal scaling in multifragmentation because it has a simple Hamiltonian and lends itself to a thermal treatment with nontrivial results [4].

Thermal scaling is the linear dependence of the logarithm of the one-fragment probability with $1/T$ (an Arrhenius plot). It indicates that the emission probability for fragment type i has a Boltzmann dependence

$$p_i = p_0 \exp(-B_i/T) \tag{1}$$

where B_i is a barrier corresponding to the emission process. As shown in Fig. 1, this is indeed the case over a wide range of temperatures and fragment sizes. While we have shown distributions for clusters up to size $A = 100$, the trend continues for larger clusters, however statistics decrease significantly as the size of the cluster increases. This linearity extends over more than four or five orders of magnitude. It rigorously confirms the form of Eq. (1) and signifies the independent thermal formation of fragments controlled by a single size-dependent barrier.

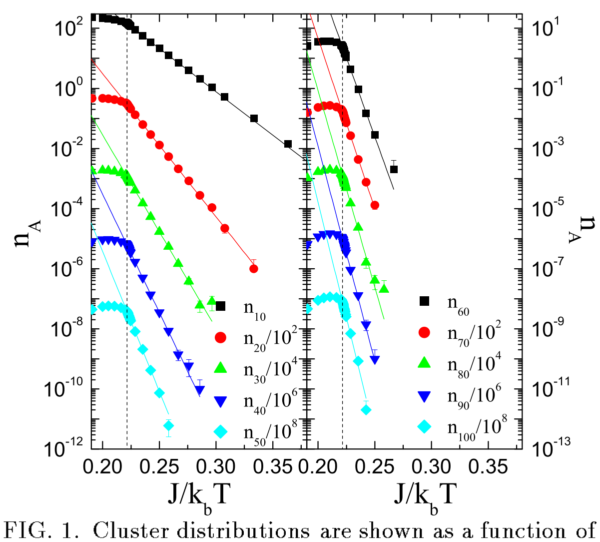

FIG. 1. Cluster distributions are shown as a function of dimensionless temperature ($T_{scaled} = k_B T/J$). Statistical error bars are shown when they exceed the size of the data point.

Thermal scaling can be found in Fisher's formula for the cluster abundance as a function of cluster size and of temperature [5,6]. In the coexistence region, the chemical potentials of the liquid and gas phases are equal and the Fisher's model predicts the cluster distribution:

$$n_A(T) = q_0 A^{-\tau} \exp(-c_0 A^{\sigma} \varepsilon/T) \tag{2}$$

with $\varepsilon = (T_c - T)/T_c$, c_0 and σ are determined from the extracted barriers and τ a critical exponent related to the topology of the system. Therefore, a graph of the scaled cluster distributions $(n_A(T)A^{\tau}/q_0)$ as a function of $\varepsilon A^{\sigma}/T$ should make the distributions of all cluster sizes collapse onto a single curve.

The nearly perfect collapse (see Fig. 2) below the critical temperature extends over six orders of magnitude for the broadest range of cluster sizes and it is perfectly linear. Therefore the three-dimensional Ising model and fluids belong to the same class of universality and can be described by Fisher's droplet model. The Ising clusters constructed here can be properly thought of as "vapor" in equilibrium with the "liquid" percolating cluster. The fact that both the three-dimensional Ising model and the experimental nuclear multifragmentation data obey the same scaling predicted by Fisher's droplet model indicates that nuclear multifragmentation can indeed be identified as the clustering in a nuclear vapor in equilibrium with the nuclear liquid.

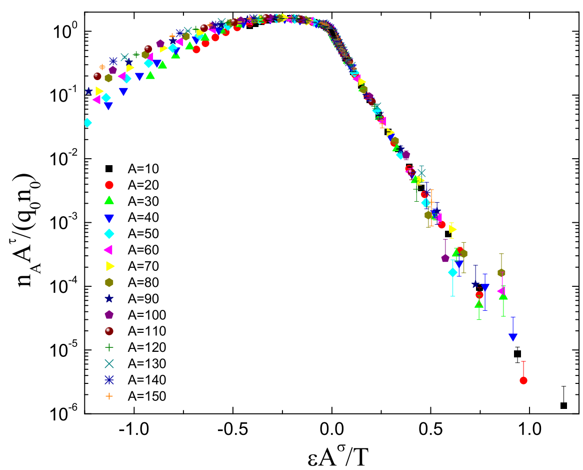

FIG. 2. Scaling behavior of cluster distributions.

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